THEORETICAL ANALYSIS OF SHOCK WAVE ANNIHILATION WITH MHD FORCE FIELD

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ABSTRACT

The annihilation of the shock waves in supersonic flows around flat bodies is analysed through one-dimensional and two-dimensional steady state studies. The main idea is that the MHD force field must act on the flow in order to keep the characteristic lines parallel each other outside the interaction region. The theoretical developments lead to an "anti shock" criterion. Numerical flow simulations, associated to shock tube experiments, are presented, corresponding to a convergent channel and a thin wing shaped body.

INTRODUCTION

Some quite recent works [1] [2] [3] [4] [5] have shown that shock waves could be cancelled by a suitable MHD force field in supersonic gas flows. The original idea, introduced by J.P.Petit, is the following : The force field **JxB** must modify the characteristic pattern of the flow in order to prevent the mutuel crossing of the characteritic lines of the same family. In other terms, one must achieve a local parallelism of the lines. In effect, in the birthplaces of the shock waves, pressure waves, following the characteristic lines, which are solutions of the hyperbolic characteristic system, tend to focus [6] [7]. If this focussing is prevented, the shocks will not occur. MHD in gas requires some electrical conductivity. Then we will get [8] :

Lorentz force field $\mathbf{F} = \mathbf{J} \times \mathbf{B}$,

Joule effect $\frac{J^2}{\sigma}$, due to the high current density J (order of magnitude 10^6 A/m^2) [9] [10]

Hall effect :

$$\beta = \frac{e^- B}{m_e \nu_e} \qquad \text{with :}$$

e^- electron charge
m_e electron mass
ν_e electron-gas collision frequency

then :

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \qquad \text{with} \quad \sigma = \frac{\sigma_0}{1+\beta^2} \begin{vmatrix} 1 & -\beta \\ \beta & 1 \end{vmatrix}$$

where **VxB** is the induced field.

We have to take account of the real gas effect in the plasma. The specific heats ratio γ decreases and tends to unity when the ionisation fraction a grows.
For high interaction parameters values the Joule effect may produce the thermal blockage of the flow. We have shown that it could be avoided [3] if :

$$K = \frac{E}{VB} << \frac{1+Zt}{\gamma-1} + 1 = KL$$

or :

$$N = \frac{\sigma_0 B^2 L (Zt+1)}{\rho V (\gamma-1)(1+\beta^2)} >> 1$$

with
B Magnetic field
σ_0 electrical conductivity
ρ Gaz density
V Velocity of the flow.
L Characteristic length

Real effect $Zt = 1/2\,\alpha(1+\alpha)$
for singly ionized argon plasma [11]
$Zt = 0$ for a perfect gas

To avoid the therrmal blockage it is not usefull to increase the magnetic field B whence $B^2/(1+\beta^2)$ tends asymptotically to $(1/\mu_e^2)$
$\mu_e = \frac{e^-}{m_e \nu_e}$ is the electronic mobility.
The ratio $B^2/(1+\beta^2)$ remains moderate.

We can consider a two temperature plasma where ($T_e > T_g$). This non-equilibrium effect increases the ionisation a and the electrical conductivity of the gas, and decreases the

Hall parameter value β when the electron-ion collision frequency become important. Strong non elequilibrium conditions would make possible to operate with moderate gaz temperature.

The Velikhov instability [12] appears when the local Hall parameter β is higher than its local critical value β_{cr} [12] . In a fully ionised plasma the critical value tends to 2 and is higher than in a partially ionized gas [13].

Theoretical studies have been achieved with the following plasma conditions, which can be provided by a shock tube wind tunnel [9] [10].

Gaz: Argon
Flow Mach number : 1.5
Velocity of the flow : 2500 m/s
Gas temperature : 10 000 °C
Gas pressure : 1 bar
Electrical conductivity : 3500 mhos/ m

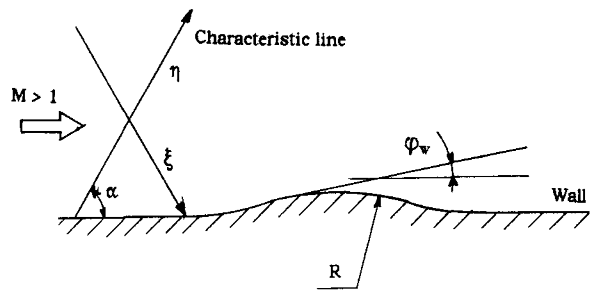
These conditions correspond to an experimentation, presently performed in the shock tube laboratory of the Rouen university, France. In order to control the characteristic lines pattern several approach have been developed :

Pure theory:Quasi one dimensional approximation
Two dimensional and axisymmetric conditions.

Numerical simulations:Two dimensional
Internal flow (in shock tube experiments)
Two dimensional external flow : wing like model.

QUASI-STEADY AND ONE DIMENSIONAL APPROACH.

Let us consider a sort of bump on a wall.



The flow can be described by Euler equations. The one dimensional flow corresponds to :

$$\frac{\partial h}{\partial \eta} = 0 \quad \Rightarrow \quad dx = \frac{\partial h}{\partial \xi} d\xi$$

We consider that all the thermodynamic quantities "h" are constant along the characteristic lines coming from the profile. We keep them parallel each other if :

$$\frac{dM}{M} = \sqrt{M^2 - 1}$$ where M is the Mach number

If we introduce real gas effects, we get :

$$\boxed{\frac{JBR}{\rho V2} = \frac{M^2}{2} + \frac{R\ \sqrt{M^2-1}\ (Zt+1)}{2\gamma M\ (\gamma+2Zt+1)} \frac{d\gamma}{\gamma}}$$

$$\boxed{\frac{dp}{p} = -\frac{Zt+1}{Zp+1} \frac{d\gamma}{\gamma+2Zt+1}} \qquad \boxed{\frac{dT}{T} = -\frac{Zt+1}{Zp+1} \frac{\gamma-1}{\gamma+2Zt+1} \frac{d\gamma}{\gamma}}$$

$$\boxed{\frac{d\rho}{\rho} = -\frac{Zt+1}{\gamma+2Zt+1} \frac{d\gamma}{\gamma}}$$

$$\boxed{\frac{dV}{V} = \sqrt{M^2-1}\ d\varphi + \frac{Zt+1}{\gamma+2Zt+1} \frac{d\gamma}{\gamma}}$$

For a perfect gas :

$$\boxed{\frac{JBR}{\rho V2} = \frac{M^2}{2}} \qquad \boxed{\frac{dp}{p} = \frac{dT}{T} = \frac{d\rho}{\rho} = 0}$$

$$\boxed{\frac{dV}{V} = \sqrt{M^2-1}\ d\varphi}$$

In order to achieve shock wave annihilation the force field must balance the pressure force all along the profile. This is similar to the classical work of Sutton [8], who determined the force field which kept constant the thermodynamic parameters (p, ρ or T) in a convergent channel.

STEADY BIDIMENSIONAL STUDY .

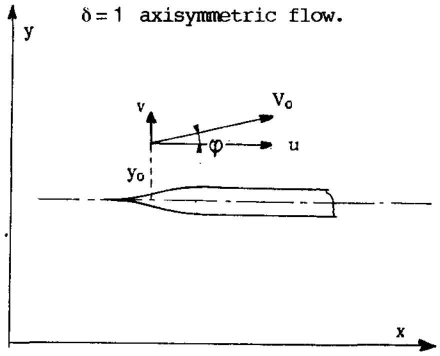
Theoretical study.

We still consider a thin profile imbeded in a supersonic flow and surrounded by a force field **JxB** We take account of real gas effects. The Hall parameter is supposed to be neglectible. Then we get the following system for a bidimensional geometry or axisymmetric geometry :

Continuity :

$$a^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} + \frac{\delta V_o}{y_o}\right) + \frac{\partial p}{\partial x} = \frac{J^2 a^2}{\sigma C_p T}$$

with $\delta = 0$ bidimensional flow
$\delta = 1$ axisymmetric flow.



Momentum

$$u\frac{\partial u}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x} = J_y B = F_x$$

$$u\frac{\partial v}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial y} = -J_x B = F_y$$

Energy equation (C1):

$$\boxed{\frac{dM}{M}=\frac{dV}{V}(M^2\frac{\gamma-1}{2}+1)-(\frac{J_yB}{\rho V^2}+\frac{J^2}{\sigma\rho V^3})M^2\frac{\gamma-1}{2}-dx}$$

The associated hyperbolic characteristic system is :

$$\boxed{dV = -\frac{dp}{\rho V} + \frac{J_yB}{\rho V}dx} \quad \text{(C2)}$$

equation (C3) :

$$\boxed{\frac{\partial P}{\partial \eta}+\frac{\partial \varphi}{\partial \eta} = -\frac{F_{\perp\eta}}{\rho V^2}+\frac{J^2\sin\alpha}{\sigma\rho V C_p T}-\frac{\delta\sin\alpha\sin\varphi}{y_0} = 2\frac{\partial\mu}{\partial\eta}}$$

equation (C4) :

$$\boxed{\frac{\partial P}{\partial \xi}+\frac{\partial \varphi}{\partial \xi} = -\frac{F_{\perp\xi}}{\rho V^2}+\frac{J^2\sin\alpha}{\sigma\rho V C_p T}-\frac{\delta\sin\alpha\sin\varphi}{y_0} = 2\frac{\partial\lambda}{\partial\xi}}$$

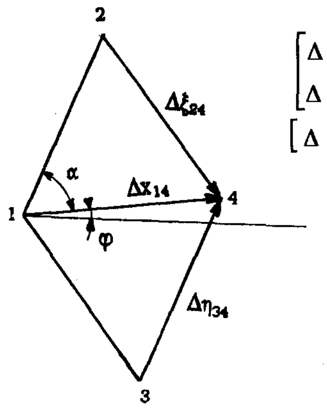
with :

$dP = \frac{\sin\alpha\cos\alpha}{\gamma p}dp$ P : Buseman number

$F_{\perp\eta} = \sin\alpha\, F_x - \cos\alpha\, F_y = F_\xi \sin 2\alpha$

$F_{\perp\xi} = \sin\alpha\, F_x + \cos\alpha\, F_y = F_\eta \sin 2\alpha$

This system (C1), (C2), (C3), (C4) makes possible to compute the local thermodynamic parameters in the flow.



$$\begin{bmatrix}\Delta\xi_{24}\\ \Delta\eta_{34}\end{bmatrix} \Rightarrow P_4 \text{ and } \varphi_4$$

$$[\Delta x_{14}] \Rightarrow V_4 \text{ and } M_4$$

If the interaction area, the complete system of equations (C1), (C2), (C3), (C4) is used. Elsewhere we shift to simplified equations system, corresponding to J = 0 .

Anti-shoch criterion and determination of the interaction area :

As a boundary condition we impose the characteristic lines to be strictly parallel, out of the interaction area. Then we get (see figure 1) :

$$\boxed{\frac{JBh}{\rho V^2} = 2\frac{\sin\alpha}{\sin(\alpha-\theta)}(\varphi_W + \frac{J^2h}{\sigma\rho V C_p T} - \delta\frac{h}{R})}$$

with :
h interaction distance
θ force field direction in the interaction area.
φ_W wall angle

From the force field **J x B** we can derive the interaction distance h.

$\theta=\alpha$: one dimensional hypothesis .

$$\frac{JBh}{\rho V^2} = \infty$$

$\theta=\alpha-\frac{\pi}{2}$: F perpendicular to the ascending characteristic lines.(minimum interaction distance h)

$$\frac{JBh}{\rho V^2} = 2\frac{\varphi_W}{M}$$

$\theta=0$: F parallel to the x-axis

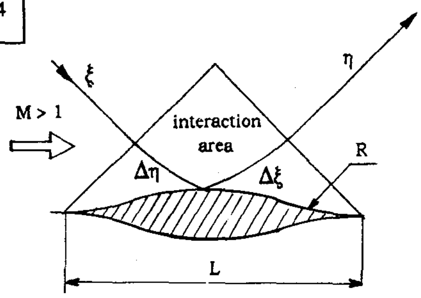
$$\frac{JBh}{\rho V^2} = 2\varphi_W$$

If we consider a wing-like thin object the interaction field must be confined in the immediate vicinity of the model, which requires a strong enough force field :

$$\Delta\eta=\frac{L}{2\sin\alpha}=\frac{LM}{2} \Rightarrow \frac{JBL}{\rho V^2} > 4\varphi_W\frac{\sin\alpha}{\sin(\alpha-\theta)}$$

$\frac{L}{\varphi_W}=R$ Consider a force field parallel to the x ($\theta=0$) axis, we get :

$$\boxed{\frac{JBR}{\rho V^2} > 4}$$



Such as it becomes possible to determine the order of magnitude of the force field, corresponding to the geometry of the object and to the inlet gas parameters of the flow.

Numerical results.

The characteristic method, as presented in the paragraph 3.1, has been applied to different peculiear configurations, with the following assumptions :

Bidimensional flow
Neglectible Joule effect
Neglectible Hall effect
Neglectible induced field VXB effect.

At first we computed the flow in a convergent nozzle. In the figure 2 we see, with a zero JXB field, how the characteristic cross each other . The place where the characteristic focuss shows the birthplace of the shock. Then a convenient JXB force field is introduced. The electrodes are located on the walls, as shown in the figure 4. The magnetic field is perpendicular to the plane of the figure. The electric field pattern is shown in the figure 3. In the figure 4 we see that the crossing of the characteristic may be avoided, such as no shock occurs. Experimental confirmation could be obtained in a shock tube wind tunnel. Of course it would correspond to short duration (quasi steady) experiments

In a second step we have computed the flow in the vicinity of a thin wing-like model. In the figure 5 the force field is zero and the focussing of the characteristics shows the birthplaces of the shocks. The empirical shaping of the force field required a great number of numerical computations. The results are presented in the figures 6, 7, 8. The figure 6 shows the shockless characteristic pattern (without any focussing). The electric field is produced by two sets of electrodes, as shown in the figure 7. The figure 8 shows the force field, shaped by a non homegeneous magnetic field distribution. The B-field is perpendicular to the plane of the figure. In the portion of the flow , close to the leading edge of the model, the gas must be accelerated, for the wall behaves as a convergent. Then it must be slowed down, in order to prevent the divergency of the characteristics (classical expansion fan) . In this region the wall can be assimilated to a divergent. In the vicinity of the end of the profile the flow must be accelerated again, for this last part the wall bahaves like a convergent and tends to slow down the flow.

CONCLUSION.

Shock wave cancellation is theoretically possible if the force field JxB can be shaped conveniently. The Lorentz force must balance the effect of the on the gas velocity. In the converging sections the gas must be accelerated and vice-versa. The results of the numerical simulations, closely associated to shock tube conditions, shows that experimental demonstration should be possible. Previous free surface water experiments gave positive results [14] . In the future the use of supraconducting magnets could make such experiments easier. The interaction parameter grows with B For high B experiments the Hall effect would become important. Then different geometry should be considered, corresponding to axially symmetric objects. Low β MHD generator are linear but high β MHD generators have disk shaped nozzles.

We think that supersonic flight could be achiedved, without shock waves. This new field should be explored and would make possible to cruise at high velocity in dense air, at low altitude and high Mach number. The shock wave system goes with strong thermal effects, strong drag increase and strong mechanical efforts. Comparing to conventional systems, MHD flight in air could be more efficient.

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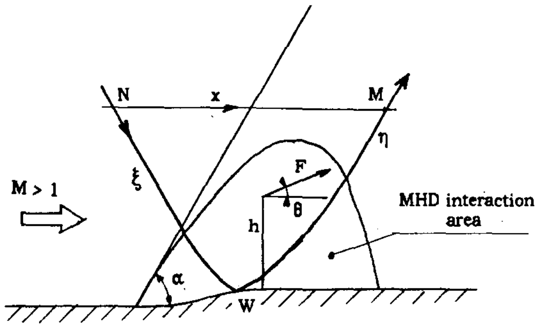

Figure 1 : Localisation of the force field near the wall.

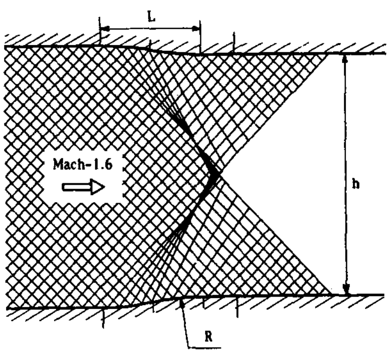

Figure 2 : Characteristic pattern in a convergent nozzle of shock tube. Shock wave occurance.

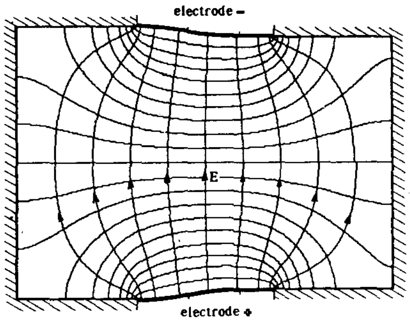

Figure 3 : Electric field in the convergent nozzle.

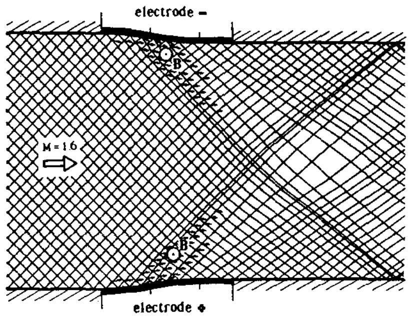

Figure 4 : Characteristic pattern with HMD force field. Annihilation of the shock wave

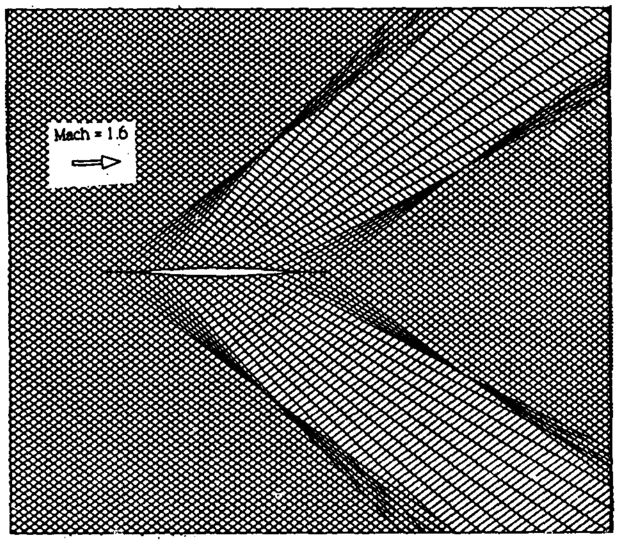

Figure 5 : Characteristic pattern around a flat body. Shock wave occurance.

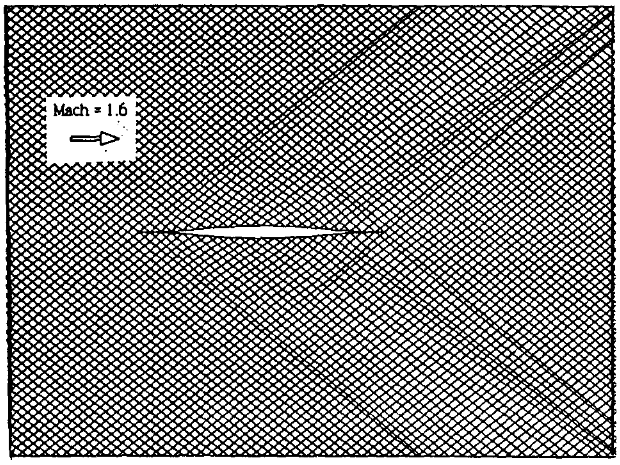

Figure 6 : Characteristic pattern with MHD force field around a flat body. Annihilation of the shock waves.

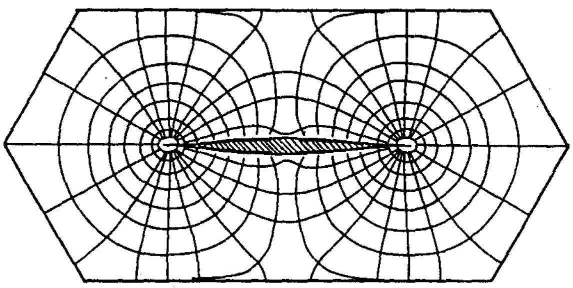

Figure 7 : Electric field computation around the wing shaped body.

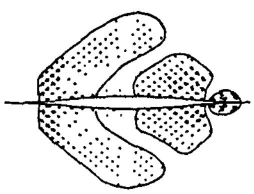

Figure 8 : MHD force field allowing shock wave annihilation.